Optimizing Matrix Multiply using PHiPAC: a Portable, High-Performance, ANSI C Coding Methodology

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Presentation Outline

1. The PHiPAC Goal
2. Methodology
3. Fast Matrix-Multiply Code
4. Matrix-Multiply Benchmark Results
5. Future Work.
The PHiPAC Goal

Automatically and portably produce a high-performance matrix-vector library maintain BLAS compatibility.

In general, ways to produce high performance code might include:

- Fixed assembly code
- High-level language: Fixed Code, Optimizing Compiler
- High-level language: Manually-tuned code, Optimizing Compiler
The PHiPAC Methodology

The PHiPAC strategy consists of 3 components:

1. Our C code, written according to special guidelines, is easy for a compiler to optimize we rely only on good instruction selection, scheduling, and register allocation.

2. Rather than code a routine by hand either in assembly or a high-level language, we write parameterized C code generators.

3. We write scripts that search the parameter space to find the optimal parameters for a given compiler and microarchitecture.
PHiPAC Coding Conventions

The following summarizes the PHiPAC code generation guidelines:

1. Name registers as locals to remove false dependencies
2. Reduce memory bandwidth demands by using local variables
3. Use base + constant offset addressing mode to avoid unnecessary pointer updates
4. Expose independent operations to hide FPU latency
5. Attempt to balance 1 multiply, 1 add, and 1-2 memory ops
6. Exploit locality, structure for unit stride and data reuse
7. Use integer addition for multiply where possible
8. Minimize branch costs where possible
9. Manual loop unrolling and software pipelining to expose ILP
Name regs as locals removing false dependencies

Instead of:

\[
\begin{align*}
a[i] &= b[i] + c; \\
a[i+1] &= b[i+1] \times d;
\end{align*}
\]

we produce:

\[
\begin{align*}
\text{float } f1, f2; \\
f1 &= b[i]; \\
f2 &= b[i+1]; \\
a[i] &= f1 + c; \\
a[i+1] &= f2 \times d;
\end{align*}
\]
Reduce memory bandwidth demands via local vars

Instead of:

```c
while (...) {
    *res++ = filter[0]*signal[0] +
            filter[1]*signal[1] +
            filter[2]*signal[2];
    signal++;  }
```

we produce:

```c
float f0,f1,f2;
f0=filter[0];f1=filter[1];f2=filter[2];
while ( ... ) {
    *res++ = f0*signal[0]
             + f1*signal[1] + f2*signal[2];
    signal++;  }
```
Use base + constant offset addressing mode

Instead of:

\[
\begin{align*}
f0 &= *r8; \quad r8 += 4; \\
f1 &= *r8; \quad r8 += 4; \\
f2 &= *r8; \quad r8 += 4;
\end{align*}
\]

we produce:

\[
\begin{align*}
f0 &= r8[0]; \\
f1 &= r8[4]; \\
f2 &= r8[8]; \\
r8 &= r8 + 12;
\end{align*}
\]
Integer addition for mult where possible

Instead of:

```c
for (i=...) {row_ptr=&p[i*col_stride];...}
```

we produce:

```c
for (i=...) {...row_ptr+=col_stride;}
```
Minimize branch costs wherever possible

Instead of:

```c
for (i=0,a=start_ptr;i<ARRAY_SIZE;i++,a++)
    { ... }
```

we produce:

```c
end_ptr = &a[ARRAY_SIZE]; a = start_ptr;
do { ... a++; } while (a != end_ptr);
```
Exposé ILP via loop unrolling/software pipelining

For example:

```c
float f0, f1, f2, s0, s1, s2;
f0 = filter[0]; f1 = filter[1]; f2 = filter[2];
s0 = signal[0]; s1 = signal[1]; s2 = signal[2];
*res++ = f0*s0 + f1*s1 + f2*s2;
do {
  signal += 3;
  s0 = signal[0]; res[0] = f0*s1 + f1*s2 + f2*s0;
  s1 = signal[1]; res[1] = f0*s2 + f1*s0 + f2*s1;
  s2 = signal[2]; res[2] = f0*s0 + f1*s1 + f2*s2;
  res += 3;
} while (...);
```
Code Generators

A *Generator* is a program that, given a set of parameters, outputs C code according to our guidelines. Our matrix multiply generator, for example, is called as follows:

- `mm_gen -cb 2 6 2 -cb 24 10 4`

Our generators have several advantages over hand or assembly coded routines:

1. It is fairly easy to benchmark a large chunk of the routine’s design space so near peak performance is more likely to be found. A generator can be *performance portable*.

2. The routine’s development cost is amortized over a large number of platforms.

3. We have found producing a generator easier than explicit hand coding and tuning.
Search Engines

We write scripts that search a generator’s entire parameter space benchmarking the resulting routines to find an optimal parameter set for a particular system. The use of search scripts can answer several questions:

- What is the fastest routine for a given workload (i.e., set of matrix sizes and their relative frequencies) under a particular compiler/machine combination?

- What is the fastest general routine across a wide range of different matrix sizes and/or different compiler/machine combinations.
Matrix Multiply Parameter Definitions
Matrix-Multiply Code Generator

The PHiPAC matrix-multiply code generator produces code for:

- The operation \( C = \alpha op(A) op(B) + \beta C \)
- Arbitrary register and cache block sizes.
- Arbitrary levels of cache blocking depth.
- Several software pipelining options: [LMA], [LM][A], [L][MA], [L][M][A]
- Different transpositions, precisions, etc.
- Called as `mm_gen -cb M0 K0 N0 [ -cb M1 K1 N1 ] ...`
Matrix-Multiply Parameter Search

Our matrix multiply search script searches the register blocking and the L1 and L2 cache-blocking parameter space.

- Register Block Search: Search all register blocking sizes within limits.
- L1-cache Block Search: Currently, we search a small neighborhood near the $D \times D$ square case, where $3D^2 = \text{L1-size}$.
- L2-cache Block Search: Similar to L1-cache search.
Performance on IBM RS/6000-590

![Graph showing performance on IBM RS/6000-590 with different computing libraries, including PHiPAC, ESSL DGEMM, and FORTRAN, 3 nested loops.

- PHiPAC
- ESSL DGEMM
- FORTRAN, 3 nested loops

MFLOPS on the y-axis, Square matrix size on the x-axis.
Performance on HP 712/80i

- PHiPAC
- Vendor DGEMM
- FORTRAN, 3 nested loops

Square matrix sizes

MFLOPS

0 10 20 30 40 50 60 70 80

0 50 100 150 200 250 300
Preliminary Performance on SGI R10k Octane

![Graph showing performance of different implementations: Vendor DGEMM, PHiPAC, and FORTRAN, 3 nested loops. The graph plots MFLOPS against square matrix sizes.]
Preliminary Performance SGI R8k Power Challenge

![Graph showing performance comparison between Vendor DGEMM, PHiPAC, and FORTRAN with 3 nested loops. The x-axis represents square matrix sizes, and the y-axis represents MFLOPS.]
Preliminary Performance on SUN Ultra-1/170

![Graph showing performance comparison between PHiPAC, Sun Performance Library, and FORTRAN, 3 nested loops.](image)

- Mflops (Million Floating Point Operations Per Second) on the y-axis.
- Square matrix sizes on the x-axis.
- PHiPAC, Sun Performance Library with `-xlic_lib=sunperf`, and FORTRAN, 3 nested loops are compared.

This graph illustrates the performance metrics for different matrix sizes, showing the efficiency of each implementation.
Future Work

- A better strategy for producing performance-portable L1 and L2 cache blocked code.
- Generators for other routines such as FFT, and the other BLAS operations.
Summary and Status

- We can write portable high-performance matrix multiply code in ANSI C using careful coding, parameterized generators, and search scripts.
- The current distribution is available on the web at: http://www.icsi.berkeley.edu/~bilmes/phipac.
- A new distribution with both faster matrix-multiply code and search will be available at the end of this summer.