1. [Oppenheim/Schafer Problem #4.15] We note that $x[n] = x[n]$ (upto multiplication by a constant) if $X(e^{j\omega}) = 0$ for $\omega / 2 \leq |\omega| \leq \pi$. For (a), $X(e^{j\omega}) = 0$ for $|\omega| > \pi/4$, and therefore there is no aliasing. For part (b), $X(e^{j\omega})$ has impulses at $\pm \pi/2$, and so aliasing occurs. For part (c), note that $X(e^{j\omega})$ can be obtained by convolving $\text{rect}(-\pi/8, \pi/8)$ with itself, (multiplication in time domain = convolution in frequency domain). Therefore, $X(e^{j\omega}) = 0$ for $|\omega| > \pi/4$. Therefore, no aliasing occurs.

2. [Oppenheim/Schafer Problem #4.19] We need $T \leq \pi/\Omega_0$.

3. [Oppenheim/Schafer Problem #4.21] Note that the $X_c(j\Omega)$ has frequency components only in the range $[\Omega_1, \Omega_2]$. Therefore, to recover the signal, we can use a (complex) ideal bandpass filter to filter out the rest of the frequency spectrum, and hence we do not care what happens there.

We now show that $\Omega_3 = \Delta \Omega$ is the smallest frequency at which one can sample the signal, without aliasing. We consider 2 cases.

(a) Suppose that the sampling frequency $\Omega_3 < \Delta \Omega$. Then
\[ TX_c(\Omega_3) = \sum_{k=-\infty}^{\infty} X_c(j(\Omega_3 - k\Delta \Omega)) = X_c(\Omega_3) + X_c(\Omega_3 - \Omega_3) + \sum_{k \neq 0} X_c(j(\Omega_3 - k\Omega_3)) \]
Now, for $\Omega_3 < \Delta \Omega$ we have $\Omega_1 < \Omega_2 - \Omega_3 < \Omega_3$, Therefore, both $X_c(j(\Omega_2 - \Omega_3))$ and $X_c(j\Omega_3)$ are non-zero, and hence aliasing occurs.

(b) Now, suppose that $\Omega_3 > \Delta \Omega$. Then, we have
\[ TX_c(\Omega_3) = \sum_{k=-\infty}^{\infty} X_c(j(\Omega_3 - k\Omega_3)) \]
Now, note that for any $k > 0$, and $\Omega \in [\Omega_1, \Omega_2]$, we have
$\Omega - k\Omega_3 < \Omega_1 < \Omega_2 - k\Delta \Omega < \Omega_2 - \Delta \Omega < \Omega_1$
and so $X_c(j(\Omega - k\Omega_3)) = 0$. Similarly, for any $k < 0$ and $\Omega \in [\Omega_1, \Omega_2]$, we have $\Omega - k\Omega_3 > \Omega_1 - k\Delta \Omega > \Omega_1 - \Delta \Omega > \Omega_2$
and so $X_c(j(\Omega - k\Omega_3)) = 0$.
Therefore for all $k \neq 0$, and $\Omega \in [\Omega_1, \Omega_2]$, we have $X_c(j(\Omega - k\Omega_3)) = 0$. Therefore, $TX_c(\Omega) = X_c(\Omega)$, and so the signal can be recovered by using a (complex) bandpass filter, which filters the frequency range $[+\Omega_1, +\Omega_2]$

4. [Oppenheim/Schafer Problem #4.23] We have $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T_1$. Therefore, we are sampling at the Nyquist rate or higher. So there will be no aliasing. In particular, we can reconstruct the signal by sinc interpolation
\[ x_c(t) = \sum_{n \in \mathbb{Z}} x[n] \frac{\sin(\pi(t - nT_1)/T_1)}{\pi(t - nT_1)/T_1} \]
where $x[n] = x_c(nT_1)$. Now, the output $y_c(t)$ is given by
\[ y_c(t) = \sum_{n \in \mathbb{Z}} x[n] \frac{\sin(\pi(t - nT_1)/T_2)}{\pi(t - nT_1)/T_2} \]
Therefore,
\[ x_c(T_1 T_2 / T_2) = \sum_{n \in \mathbb{Z}} x[n] \frac{\sin(\pi(t - nT_1)/T_1)}{\pi(t - nT_1)/T_1} \]
\[ = \sum_{n \in \mathbb{Z}} x[n] \frac{\sin(\pi(t - nT_1)/T_2)}{\pi(t - nT_1)/T_2} \]

5. [Oppenheim/Schafer Problem #4.26] If $x[n] \leftrightarrow X(e^{j\omega})$, then since $x_d[n]$ is obtained by downsampling $x[n]$, we have
\[ x_d[n] \leftrightarrow X_d(e^{j\omega}) = \frac{1}{M} \sum_{m=-M+1}^{M-1} X(e^{j(\omega - \frac{2\pi}{M})}) \]
Further, since we can consider $x[n]$ the result of upsampling $x_d[n]$, we have
\[ x[n] \leftrightarrow X_s(e^{j\omega}) = X_d(e^{j\omega}) \]
Therefore, if the upsampling/downsampling factors are $M = 3$, we have
\[ X_d(e^{j\omega}) = X(e^{j\frac{\omega}{3}}) + X(e^{j\frac{\omega+2\pi}{3}}) + X(e^{j\frac{\omega+4\pi}{3}}) \]
\[ = \frac{X(e^{j\pi/3}) + X(e^{j\pi+2\pi/3}) + X(e^{j\pi+4\pi/3})}{3} \]
(a) Figures 1 and 2 show $X_d(e^{j\omega})$ and $X_s(e^{j\omega})$ for $M = 3$ and $\omega_H = \pi/2$. The dotted lines show the 3 components, while the solid line shows the sum.
(b) Figures 3 and 4 show $X_a(e^{j\omega})$ and $X_A(e^{j\omega})$ for $M = 3$ and $\omega_H = \pi/4$.

6. [McClellan et al. #1.1, Page 36] Because of aliasing, a frequency of 7525 + 125$k$ is indistinguishable from $-475 + 125k$. In the range $[-4000, 4000]$, greater absolute values correspond to “higher frequencies”. Since the frequencies 7525, 7650, 7775, 7900 correspond to $-475, -350, -225, -100$, the frequencies are “decreasing”. The frequencies 32100, 32225, 32350, 32475 correspond to 100, 225, 350, 400, an increasing set of frequencies.

7. [McClellan et al. #1.2, Page 32] The instantaneous frequency of the sampled signal will appear to be small (or close to 0) whenever the instantaneous frequency of the signal is close to a multiple of 8000. This happens when $\mu t$ is of the form $4000 + k \cdot 8000$. This takes place for $t$ near 0, 0.0067, 0.02, 0.0333, 0.0467.