1. **[Oppenheim/Schafer Problem #2.47]**

   \[ y[n] = x[n] + 2x[n-1] + x[n-2] = x[n] + (\delta[n] + 2\delta[n-1] + \delta[n-2]). \]

   Therefore, \( h[n] \), the impulse response is \( \delta[n] + 2\delta[n-1] + \delta[n-2] \). The system is stable because \( \sum_{k=-\infty}^{\infty} |h[k]| = 4 < \infty \).

   \[
   H(e^{j\omega}) = 1 + 2e^{-j\omega} + e^{-2j\omega} \\
   = 2e^{-j\omega}(0.5e^{j\omega} + 1 + 0.5e^{-j\omega}) \\
   = 2e^{-j\omega}(1 + \cos(\omega)).
   \]

   \[ |H(e^{j\omega})| = 2(\cos(\omega) + 1), \text{ and } \arg H(e^{j\omega}) = -\omega. \]

2. **[Oppenheim/Schafer Problem #2.59]**

   \[ R_x[n] = \sum_{k=-\infty}^{\infty} x^*[k]x[n+k] \]

   \[ = \sum_{r=-\infty}^{\infty} x^*[\text{}\pm r\text{]}x[n-r] \]

   Substitute \( r = -k \)

   \[ = x^*[-n]x[n] \]

   Therefore, \( g[n] = x^*[-n] \).

   For part (b), note that \( x^*[-n] \sim X^*(e^{j\omega}) \). Hence \( R_x(e^{j\omega}) = X(e^{j\omega})x^*(e^{j\omega}) = |X(e^{j\omega})|^2 \).

3. **[Oppenheim/Schafer Problem #2.60]**

   \[ x_2[n] = \sum_{k=-\infty}^{\infty} x[k] = y_2[n] \]

   Therefore \( y_2[n] = \sum_{k=-\infty}^{\infty} y[k] \).

   For part (b), note that \( \delta[n] = \sum_{k=-\infty}^{\infty} x[n-k] \).

   Therefore by linearity, \( h[n] = \sum_{k=-\infty}^{\infty} y[k] \).

   One thing that you should note however is that the solution is not unique, even though it seems to be. The reason is that we could have potentially formed \( \delta[n] \) by other combinations of the \( x[n] \) as well. In general, \( h[n] \) satisfies the relation

   \[ h[n-1] + h[n] = y[n] \]

   This leads to a unique solution if we make additional assumptions (such as causality, FIR-ness etc.). Otherwise, we get a solution that is unique up to the addition of a constant.

4. **[Oppenheim/Schafer Problem #2.62]**

   In this question, you were asked to use the definition of causality to show that \( h[n] \neq 0 \) for some \( n < 0 \) if and only if the system is causal. A system is causal if whenever \( \{x_1[n]\} \rightarrow \{y_1[n]\} \) and \( \{x_2[n]\} \rightarrow \{y_1[n]\} \) satisfy \( x_1[n] = x_2[n] \) for all \( n \leq n_0 \), then \( y_1[n] = y_2[n] \) for all \( n \leq n_0 \).

   For the forward direction, assume that \( h[n_0] \neq 0 \) for some \( n_0 < 0 \). Let \( x_1[n] = \delta[n+n_0] \).

   Then \[
   y_1[0] = \sum_{k=-\infty}^{\infty} x_1[k]h[0-k] = x[-n_0]h[0] = h[n_0] \neq 0
   \]

   Let \( x_2[n] = 0 \) for all \( n \in \mathbb{Z} \). Then by a previous homework problem, \( y_2[n] = 0 \) for all \( n \in \mathbb{Z} \). Therefore, we have \( x_1[n] = x_2[n] \) for all \( n < -n_0 - 1 \). Now, \( -n_0 - 1 \geq 0 \). However, \( h[n_0] = y_1[0] = y_2[0] = 0 \). Therefore, the system is not causal.

   For the converse, note that \( y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \). So, if \( x_1[n] = x_2[n] \) for all \( n \leq n_0 \), then \( x_1[n-k] = x_2[n-k] \) for all \( n \leq n_0, k \geq 0 \). Hence

   \[ y_1[n] = \sum_{k=-\infty}^{\infty} h[k]x_1[n-k] = \sum_{k=-\infty}^{\infty} h[k]x_2[n-k] \]

   for \( n \leq n_0 \). Therefore the system is causal.

5. **[Oppenheim/Schafer Problem #2.66]**

   \[ E(e^{j\omega}) = H_1(e^{j\omega})X(e^{j\omega}) \]

   \[ F(e^{j\omega}) = E(e^{-j\omega}) = H_1(e^{-j\omega})X(e^{-j\omega}) \]

   \[ G(e^{j\omega}) = H_1(e^{j\omega})F(e^{j\omega}) = H_1(e^{j\omega})H_1(e^{-j\omega})X(e^{j\omega}) \]

   \[ Y(e^{j\omega}) = G(e^{j\omega}) = H_1(e^{j\omega})H_1(e^{-j\omega})e^{j\omega}X(e^{j\omega}) \]

   Therefore, \( H(e^{j\omega}) = H_1(e^{-j\omega})H(e^{j\omega}) \). Therefore \( h[n] = h_1[-n] \ast h_1[n] \).

6. **[Oppenheim/Schafer Problem #2.81]**

   Because \( s \) and \( e \) are uncorrelated, we have \( E\{s[n]e[m]\} = 0 \) for all \( n, m \). Hence \[
   E\{y[n]|y[n+m]\} = E\{s[n]e[n+s[n+m]]e[n+m]\} = E\{s[n]s[n+m]e[n+m]\}
   \]

   Using the fact that \( s, e \) are uncorrelated, we get

   \[ = E\{s[n]s[n+m]\} E\{e[n]e[n+m]\} \]

   \[ = \sigma_s^2 \sigma_e^2 \delta[m] \]

   because \( f[m] \delta[m] = f[0] \delta[m] \).