4.1 Problems from the Text


4.2 Additional Problems

**Problem 1:** Let $x_{1:n}$ be an arbitrary length-$n$ binary string (i.e., so that $x_i \in \{0, 1\}$ for all $i$). Come up with as many upper bounds on $K(x_{1:n}|n)$ as you can, but in each case try to make the upper bounds as small as possible. Of the set of upper bounds that you have found, which is the best?

**Problem 2:** In class and in the text, we defined the amazing incredible and unknowable number $0 < \Omega < 1$. In this problem, you are to choose a normally unsolvable problem (it can be one from mathematics, or even any general world problem you wish you could solve), and show how that if you have $\Omega$ available to you, you can compute a (guaranteed halting) solution to this problem. Be as precise as possible, in that you give an explicit algorithm for how, when $\Omega$ is given, you can compute an answer to your problem. Argue why your algorithm is correct, and why the problem you choose is normally unsolvable. Estimate the running time of your problem if possible.

**Problem 3:** This is really book problem 14.5, but do only part a,b,c, and d (not e). For problem 14.5d, compute the probability of the sequence $\omega_1 \omega_2 \ldots \omega_n$ which is then followed by any arbitrary sequence, where

$$\Omega = \sum_{p: l(p) \text{ halts}} 2^{-l(p)}, \quad \text{and} \quad \Omega = \omega_1 \omega_2 \ldots .$$

**Problem 4:** Using the Laplace model that we outlined in class, describe the Arithmetic coding algorithm for encoding a random bit string of length $n$ with $k$ ones, given $n$ and $k$. For the case of the string of length $n = 4$, show in complete detail the intervals corresponding to all source substrings of length $1 - 4$. 
