Final – EE235
12 December 2000

Instructions:

• The test is closed book and you are allowed two 8.5×11 page of notes. No calculators are allowed.

• Show all work. Partial credit will be given for partial work; NO credit will be given for no work even if the answer is correct.

• Be sure to state all assumptions made and check them when possible.

• There are five problems on six pages, including the cover. The formula sheet will be handed out separately. A blank page is attached at the end in case you need extra space.

Honor Code:
This exam represents only my own work. I did not give or receive help on this exam.
1. (20 points)
Each of the parts below refer to the same LTI system $S$. You are given that an input of $x_1(t) = u(t + 1) - u(t)$ gives the output $y_1(t)$ illustrated below, but you are not given the impulse response $h(t)$.

(a) (8 points) Find the output $y_2(t)$ when $x_2(t) = u(t + 1) - 2u(t) + u(t - 1)$. Hint: use linearity.

(b) (7 points) Determine which of the following outputs is the step response of the system and find $A$. Explain.

(c) (5 points) Do you have enough information to find the impulse response of the system? Explain.

Yes, since we found $s(t)$ and $h(t) = \frac{d}{dt}s(t)$
2. (25 points)
An LTI system has frequency response \( H(\omega) \) illustrated below.

Find the outputs \( y_i(t) \) in the time domain for each of the inputs \( x_i(t) \) below.

(a) (8 points) \( x_1(t) = 1 + \cos(\frac{\pi}{2} t) \)

\[
y_1(t) = H(0) + \left| H\left(\frac{\pi}{2}\right) \right| \cos\left(\frac{\pi}{2} t + \angle H\left(\frac{\pi}{2}\right)\right)
= \frac{3}{2} + \frac{3}{2} \cos\left(\frac{\pi}{2} t - \frac{\pi}{3}\right)
\]

(b) (8 points) \( x_2(t) \) is the rectangle wave illustrated below with \( T_0 = 4 \).

\[
y_2(t) = H(0)C_0 + 2 |C_1||H\left(\frac{\pi}{2}\right)| \cos\left(\frac{\pi}{2} t + \angle H\left(\frac{\pi}{2}\right) + \angle C_1\right)
= \frac{3}{2} + \frac{3}{2} \cos\left(\frac{\pi}{2} t - \frac{\pi}{3}\right)
\]

(c) (9 points) Let \( x_3(t) = \delta(t + \frac{1}{3}) \). Describe the output \( y_3(t) \) qualitatively, e.g. is it a squared sinc function centered at \( t = 0 \), a sinc shifted to the left, a triangle shifted to the right, etc.?

\[
sinc \text{ squared, shifted to the right by } 1/3
\]
3. (15 points)
Consider the pole-zero plots illustrated below.

(a) (5 points) Specify all (if any) that could correspond to a causal, stable LTI system. (Explain)
   
   *only (A) – all poles must be in left half plane*

(b) (5 points) Specify all (if any) that could have an impulse response of the form $h(t) = (A + Be^{-at} \cos(\omega_0 t + \phi))u(t)$. (Explain)
   
   *only (B) – Au(t) term gives pole at zero, damped cosine gives complex poles*

(c) (5 points) Specify all (if any) that could correspond to a real-valued time signal. (Explain)
   
   *ALL – all poles and zeros are real or have conjugate pairs*
4. (25 points)
Two LTI systems $S_1$ and $S_2$ are connected in series. It is known that the overall system response is
\[ H(s) = \frac{s(s + 1)}{(s^2 + 2s + 2)(s + 2)} \]
for $\text{Re}\{s\} > -1$ and that the impulse response of $S_2$ is $h_2(t) = e^{-t} \cos(t)u(t)$.

(a) (9 points) Find the transfer function $H_1(s)$ that describes system $S_1$.

\[ H_1(s)H_2(s) = H(s) \implies H_1(s) = H(s)/H_2(s) \]

\[ H_2(s) = \frac{s + 1}{s^2 + 2s + 2} \]
\[ H_1(s) = \frac{s}{s + 2} \]

(b) (8 points) Is system $S_1$ stable? Explain.

The overall ROC is right-sided, so individual poles are right-sided and the ROC of $S_1$ contains the $j\omega$-axis. Therefore, it is stable.

(c) (8 points) Find the differential equation (LCCDE) that describes $S_1$. (If you did not get part (a), determine what the system order should be and write $H_1(s)$ and the LCCDE in terms of generic coefficients $a_i$ and $b_i$.)

\[ \frac{d}{dt} y(t) + 2y(t) = \frac{d}{dt} x(t) \]
5. (15 points)
A signal \( x(t) = e^{-at} u(t) \) is low-pass filtered with \( H_L(\omega) \) and then sampled using frequency \( \omega_s = 2\pi/T_s \).

(a) (5 points) Why is the low-pass filter needed for sampling this particular signal?

   To make the signal bandlimited, so as to avoid (or reduce) aliasing effects. In other words, eliminate or at least reduce the energy outside of the frequency range \((-0.5\omega_s, 0.5\omega_s)\).

(b) (5 points) If it was an ideal LPF, what would be the appropriate bandwidth \((W_i, i \text{ for ideal})\)? Explain.

   \[
   W_i = \frac{\omega_s}{2}
   \]

(c) (5 points) Assuming that the sampling rate is fixed, how would you change the filter bandwidth \((W_r, r \text{ for real})\) for the practical problem of having real (not ideal) filters? Specifically, compare the real filter bandwidth \(W_r\) to the ideal filter bandwidth \(W_i\).

   Since real filters don’t have sharp cut-offs, you need \( W_r < W_i \) to have fewer problems with aliasing.

End Of Exam